# Operating Characteristic Curve a trade off provider between Producer's and consumer's risk

* Dr. Ramesh R. Kulkarni

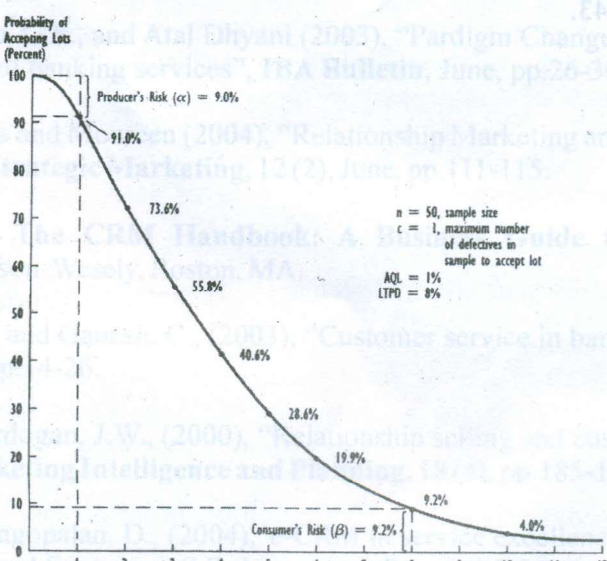
## Introduction :

Quality control has been an ever daunting task for all the manufacturers in the universe, whether called a 3 or 6 sigma organization. It has never been an easy task for the producers to control their quality to the desired one. It is an every organization's dream to be an 100% quality producing and serving in the market. But it is equally challenging to achieve it. Every producer would be aiming to sell all that he has produced and hence would be always working out on the possibilities of his product being sold. Therefore It is always desirous for him that, he would possess a machinery that can work out and tell him the possibility or probability of his goods being sold or accepted by the customer. Most of these organizations today would be looking for testing methods that are so quick and effective that products are submitted to 100 percent inspection and testing which means that, every product shipped to customers is inspected, and tested to determine whether it is of desired quality. But it is literally not possible in all the cases to do so. For some such products where 100 testing becomes uneconomical, impractical, or impossible, acceptance plans are the only sensible basis for inspecting and testing.

Over the past few decades statisticians have been trying to develop one such machinery which can serve as a very good quality control tool on which a producer can rely upon. Control charts and Acceptance sampling are the few which people have been using for the purpose. Among these tools, OC curve is a graphical tool which expresses the quality status of a sample of products whether to accept or reject. Here is an effort to understand the behavior of OC curve as a trade off between both producers and consumers risk.

## Operating Characteristic curve.

An Operating Characteristic Curve is a graphical tool to exhibit the performance of an acceptance plan . It shows how well an acceptance sampling plan discriminates between good and bad lots of production. It is a graph of Percentage defectives in lots versus the probability of accepting these lots. Fig. 1 shows a model OC curve.




* Director Bharatesh Education Trust's Global Business School Belgaum Karnataka
Email: director@betgbs.in

OC curve is a tool associated with acceptance sampling. We know that, acceptance sampling ultimately results into accepting a good lot, rejecting a bad lot, accepting a bad lot or rejecting a good lot. At the end of acceptance sampling we may commit two types of errers, either rejecting a good lot or accepting the bad lot. Hence we can always work out the chance or committing these two errors. The above fig.1 gives reference to the following four definitions

**Acceptable Quality Level (AQL):**

It is a quality level and lots of this level of quality are regarded as good, and we wish to have a high probability for their acceptance.

**Producer's risk or Type-I error ( $\alpha$ ):**

It is defined as the probability of rejecting a good lot, in other words It is the probability that, the lots of the quality level AQL will not be accepted. Usually 5% in practice.

**Lot Tolerance percent defective (LTPD):**

It is the dividing line selected between good and bad lots. Lots of this level of quality are regarded as poor, and we wish to have a low probability for their acceptance.

**Type-II error ( $\beta$ ) or Consumer's risk:**

It is defined as the probability of accepting a bad lot. In other words the probability that lots of the quality level LTPD will be accepted. Usually it will be 10% in practice.

In practice we want acceptance sampling plans that discriminate between good and bad samples or lots. Sampling plans do not provide perfect discrimination between good and bad lots. Some lots of low quality may be accepted while some lots of good quality may be rejected due to sampling errors. The degree to which a sampling plan discriminates between good and bad lots is a function of the steepness of the OC curve.

**Illustration:**

Let us consider a case where,
n= sample size,
N=Lot size,
c=largest number of defectives per sample to accept the lot,
$\pi$=percent defective in a lot coming into inspection
P(A)= Probability of accepting a lot
P(B)= Probability of rejecting the lot.
$\alpha$= producer's risk ( P(R)) at AQL)
$\beta$=consumer's risk ( P( A) at LTPD)

Where the proba $\frac{(P(x)=(100)\ (n\frac{\pi}{100})^x}{x!}e^{-n}$ es in a sample can calculated by using poisson distribution given

by P(x) = (100) $\left(\frac{50\ \frac{1}{100}}{0!}\right)^0$ /100)

For e.g P(0) = (100) $2.71828^{-50/100}$ = 60.7

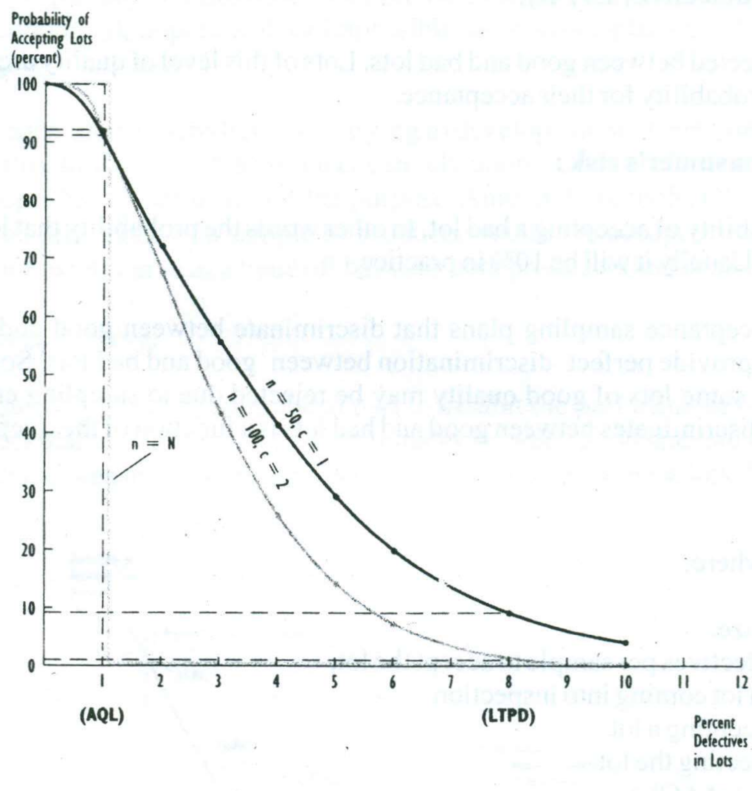
Similarly P(1)= 30.3

And therefore, P( A) = P(0)+ P(1)=91.0

| | N=50, c=1 | | N=100, c=2 | | N=N | |
|---|---|---|---|---|---|---|
| . | P(A) | P(R) | P(A) | P(R) | P(A) | P(R) |
| 0 | 100 | 0 | 100 | 0 | 100 | 0 |
| AQL = 1 | 91.0 | α= 9.0 | 92.0 | α= 8.0 | 100 | α= 0 |
| 2 | 73.6 | 26.4 | 67.7 | 32.3 | 0 | 100 |
| 3 | 55.8 | 44.2 | 42.3 | 57.7 | 0 | 100 |
| 4 | 40.6 | 59.4 | 23.8 | 76.2 | 0 | 100 |
| 5 | 28.6 | 71.3 | 12.5 | 87.5 | 0 | 100 |
| 6 | 19.9 | 80.1 | 6.2 | 93.8 | 0 | 100 |
| LTPD=8 | β =9.2 | 90.8 | β = 1.4 | 98.6 | β =0 | 100 |
| 10 | 4.0 | 96.0 | 0.3 | 99.7 | 0 | 100 |

Exihibit-1 Table showing Probability calculations for OC curve.

Fig. 2 showing the OC curves for different sample size




From the Exhibit 1 and fig 2 above, it is clear that, we will have a different OC curve for different sample size. Which is also in conformance with the properties of poison distribution. When compared to n=50, c=1, with n=100, and c=2, notice that by doubling n from 50 to 100 and doubling c from 1 to 2, we have kept the c/n ratio the same, but α has been reduced from 9.0 to 8.0 percent and β has been reduced from 9.2 percent to 1.4 percent. Thus the ability of the sampling plans to discriminate between good and bad lots is enhanced by increasing sample size. This means that, we would then reject fewer good lots and accept fewer bad lots. To further demonstrate this point consider the OC curve of the acceptance sample plan of n=N in fig 2. This curve perfectly discriminates between the good and bad lots as both α , β are equal to 0. In other words the probability of accepting a lot with 1% or less defective is 100% and probability of rejecting a lot with more than 8% defectives is 100%. But in this plan, the sample size is equal to the lot size. It means every item in the lot would have to be tested. Which is really difficult proposition. Therefore managers can reduce producer's risk and consumer's risk with an additional cost of taking larger samples.

In other words, from fig 2 it can be observed that, for the same percentage defectives as the maximum allowable for an acceptable lot, the larger the sample size, the greater is the discriminating power. Acceptance plans with smaller sample sizes reject more good lots and accept more bad lots. However it should be noted that higher the sample size , higher is the inspection costs. Therefore we must make trade off decisions when designing acceptance plans.

If perfect discrimination is required between a good lot and bad lot, say for example, in a lot of product of size N=100, units, if the quantity of defective units is less than 2% the lot is to be accepted and if the quantity of defectives is greater than 2% the lot is to be rejected then the OC curve drawn is shown in dotted lines in fig 2. This kind of discrimination is called as perfect discrimination and the curve is called as ideal oc curve.

**Limitations of OC curve.**

Since the OC curves are numerical in nature, they pose following limitations :

- Calculation of P( A) and P( R) is a bit of difficulty for a person or manager without having mathematical and statistical background.
- The curves being numerical and mathematical in nature do not cover the behavioral aspects of choosing sample. In occasions where behavioral aspects are covered OC curves fail to trade off between risks.
- The whole process of drawing OC curves and then drawing conclusions is also time consuming and can not substitute managerial expertise of the person involved.

**Bibliography**

1. Business Statistics – G.C. Beri, 3rd Edition Mc.Graw Hill edition
2. Operations Management – Norman Gaither, Greg Frazier.,South Western Cleanage Learning, India Edition
3. Modern Production and Operations Management- Edward.S.Buffa. Willey Edition
4. Website WWW. engineerlive.com